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# A classification of assembly line balancing problems

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## Abstract

Assembly lines are special flow-line production systems which are of great importance in the industrial production of high quantity standardized commodities. Recently, assembly lines even gained importance in low volume production of customized products (mass-customization). Due to high capital requirements when installing or redesigning a line, its configuration planning is of great relevance for practitioners. Accordingly, this attracted attention of plenty researchers, who tried to support real-world configuration planning by suited optimization models (assembly line balancing problems). In spite of the enormous academic effort in assembly line balancing, there remains a considerable gap between requirements of real configuration problems and the status of research. To ease communication between researchers and practitioners, we provide a classification scheme of assembly line balancing. This is a valuable step in identifying remaining research challenges which might contribute to closing the gap.

*Keywords:* Configuration of assembly lines; Assembly line balancing; Classification

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## 1. Introduction

An assembly line is a flow-oriented production system where the productive units performing the operations, referred to as stations, are aligned in a serial manner. The workpieces visit stations successively as they are moved along the line usually by some kind of transportation system, e.g. a conveyor belt.

Originally, assembly lines were developed for a cost efficient mass-production of standardized products, designed to exploit a high specialization of labour and the associated learning effects (Shtub and Dar-El, 1989; Scholl, 1999, p. 2). Since the times of Henry Ford and the famous model-T, however, product requirements and thereby the requirements of production systems have changed dramatically. In order to respond to diversified customer needs, companies have to allow for an individualisation of their products. For example, German car manufacturer BMW offers a catalogue of optional features which, theoretically, results in  $10^{32}$  different models (Meyr, 2004). Multi purpose machines with automated tool swaps allow for facultative production sequences of varying models at negligible setup costs. This makes efficient flow-line systems available for low volume assembly-to-order production

(Mather, 1989) and enables modern production strategies like mass-customization (Pine, 1993), which in turn ensures that the thorough planning and implementation of assembly systems will remain of high practical relevance in the foreseeable future.

Due to the high level of automation, assembly systems are associated with considerable investment costs. Therefore, the (re)-configuration of an assembly line is of critical importance for implementing a cost efficient production system. Configuration planning generally comprises all tasks and decisions which are related to equipping and aligning the productive units for a given production process, before the actual assembly process can start. This includes setting the system capacity (cycle time, number of stations, station equipment) as well as assigning the work content to productive units (task assignment, sequence of operations).

In light of the high practical relevance, it is not astounding that a massive body of academic literature covers configuration planning of assembly systems. In the scientific discussion, the term assembly line balancing (ALB) is used to subsume optimization models which seek to support this decision process. Since the first mathematical formalization of ALB by Salvendy (1955), academic work mainly focused on the core problem of the configuration, which is the assignment of tasks to stations. Because of the numerous simplifying assumptions underlying this basic problem, this field of research was labeled simple assembly line balancing (SALB) in the widely accepted review of Baybars (1986). Subsequent works however, more and more attempted to extend the problem by integrating practice relevant aspects, like u-shaped lines, parallel stations or processing alternatives (Becker and Scholl 2006). In spite of these efforts, which are referred to as general assembly line balancing (GALB), there seems to be a wide gap between the academic discussion and practical applications. Empirical surveys stemming from the 70s (Chase, 1974) and 80s (Schöniger and Spingler, 1989) revealed that only a very small percentage of companies were using a mathematical algorithm for configuration planning at that time. The apparent lack of more recent scientific studies on the application of ALB-algorithms indicates that this gap still exists or even has widened.

Three theoretical reasons might explain the aforementioned deficit: (i) Researchers have not considered the “true” real-world problems so far. (ii) The problems were covered, but could not be solved to satisfaction. (iii) Scientific results could not be transferred back to practical applications, e.g. because solutions for special case studies could not be extended to general problems.

Any of these reasons might result from a fundamental problem in communication, which is expressed by an inconsistent use of terms and definitions for the various types of balancing problems (Becker und Scholl, 2006). This is not only impeding the communication within the research community, but also the knowledge transfer to practice.

A first, yet decisive step to resolve this problem lies in a consistent, authoritative classification of assembly line balancing problems including all relevant constraints and objectives. A uniform classification enables practitioners to compare their individual problem settings with those covered by research and to single out suitable solution techniques. Furthermore, future research challenges can be identified by structuring the existing body of literature according

to the classification scheme. The primary aim of this article is to develop such a classification.

Apparently, the existing distinction of SALB and GALB introduced by Baybars (1986) has become insufficient to reflect the heterogeneity of GALB problems. Especially now, when the problem structure of SALB is well examined and powerful solution techniques exist, it is to be expected that future publications will mainly focus on GALB problems, some of which might show a similar problem structure as SALB, whereas others will deviate considerably. Ghosh and Gagnon (1989) extended Baybars efforts by further distinguishing special characteristics of GALB problems which had been covered by academic work at that time. However, their list lacks a systematic approach and is long outdated because a great variety of additional constraints has been introduced in the meantime. Several researchers have tried to overcome this weakness by developing individual names for their considered problem extensions, which were mostly oriented towards the existing nomenclature. Although these efforts certainly help experts in the field to recognize particular problem characteristics, it might also lead to confusion as long as no guidelines exist as to when an extension is different enough in order to receive a new label or when two extensions with individual names are to be combined. In any case, this policy hardly reveals relations between problems and therefore cannot replace a classification in structuring the literature.

Before a new classification is proposed, the subject to be classified has to be characterized unambiguously. Therefore, section 2 will start out with a description of the core problem of ALB, which will be used as a basis for the presented classification scheme. Section 3 gives a detailed explanation of those elements of ALB, which are mainly ignored or extremely simplified in the SALB formulation. This leads to the classification scheme in section 4. Then, the classification scheme is applied to structure the existing literature in section 5. Section 6 provides a first interpretation of achieved results and aims at providing hints on how to close the gap between research and practice by identifying promising areas of future research and characterizing practice relevant problem extensions which have not been covered so far.

## **2. A Definition of Assembly Line Balancing (ALB)**

In the following, we define the scope of the classification. First we characterize the SALB problem as the core decision problem in configuration planning in its very basic version. Afterwards, the basic assumptions of SALB are examined of how they have to be adopted for setting the more general assumptions of ALB. That way, a definition of ALB, the field to be classified, can be derived.

Among the family of ALB problems, the most well-known and well-studied is certainly the SALB problem. Although it might be far too constrained to reflect the complexity of real-world line balancing, it nevertheless captures its main aspects and is rightfully regarded as the core problem of ALB. In fact, vast varieties of more general problems are direct SALB extensions or at least require the solution of SALB instances in some form. In any case, it is well suited to explain the basic principles of ALB and introduce its relevant terms. A com-

prehensive review of SALB and its solution procedures is provided by Scholl and Becker (2006).

According to the underlying concept of any SALB formulation, an *assembly line* consists of  $k = 1, \dots, m$  (*work*) *stations* arranged along a conveyor belt or a similar mechanical material handling device. The *workpieces* (*jobs*) are consecutively launched down the line and are hence moved on from station to station until they reach the end of the line. A certain set of operations is performed repeatedly on any workpiece which enters a station, whereby the time span between two entries is referred to as *cycle time*. In general, the line balancing problem consists of optimally partitioning (balancing) the assembly work among all stations with respect to some objective. For this purpose, the total amount of work necessary to assemble a workpiece is split up into a set  $V = \{1, \dots, n\}$  of elementary operations named *tasks*. Tasks are indivisible units of work and thus each task  $j$  is associated with a processing time  $t_j$  also referred to as *task time*. Due to technological and/or organizational requirements, tasks cannot be carried out in an arbitrary sequence, but are subject to *precedence constraints*.

The general input parameters of any SALB instance can be conveniently summarized and visualized by a *precedence graph*. This graph contains a node for each task, node weights which equal the task times and arcs reflecting direct as well as paths reflecting indirect precedence constraints. Figure 1 shows an example precedence graph with  $n=9$  tasks having task times between 2 and 9 (time units).

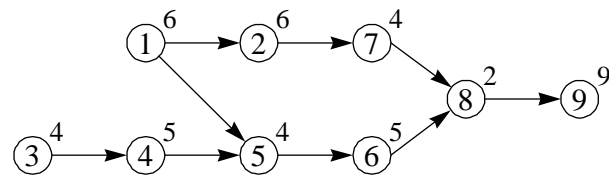


Figure 1. Precedence graph

A feasible *line balance*, i.e. an assignment of tasks to stations, has to ensure that no precedence relationship is violated. The set  $S_k$  of tasks assigned to a station  $k$  constitutes its *station load* or *work content*, the cumulated task time  $t(S_k) = \sum_{j \in S_k} t_j$  is called *station time*.

SALB further assumes that the *cycle time* of all stations is equal to the same value  $c$ . Assembly lines with this attribute are called *paced*, as all stations can begin with their operations at the same point in time and also pass on workpieces at the same rate. As a consequence, all station times of a feasible balance may never exceed  $c$ , as otherwise the required operations could not be completed before the workpiece leaves the station. Station times can however be smaller than the cycle time, in which case a station  $k$  has an unproductive *idle time* of  $c - t(S_k)$  time units in each cycle.

For the example of Figure 1, a feasible line balance with cycle time  $c=11$  and  $m=5$  stations is given by station loads  $S_1=\{1,3\}$ ,  $S_2=\{2,4\}$ ,  $S_3=\{5,6\}$ ,  $S_4=\{7,8\}$  and  $S_5=\{9\}$ .

In order to ensure high productivity, any good balance should cause as few idle times as possible. With regard to the objective function considered, SALB problems can further be distinguished (cf. Scholl, 1999, ch. 2.2) into four types: For a given cycle time  $c$ , minimizing the sum of station idle times is equal to minimizing the number of opened stations. SALB problems under this objective are called SALBP-1. Conversely, if the number of stations is given,

then minimizing the cycle time guarantees minimum idle times, which is known as SALBP-2. If both, number of stations and the cycle time, can be altered, the *line efficiency*  $E$  is used to determine the quality of a balance. The line efficiency corresponds to the productive fraction of the line's total operating time  $t_{\text{sum}}$  and is typically defined as  $E = t_{\text{sum}} / (m \cdot c)$ . As the total idle time is equal to  $t_{\text{sum}} - (m \cdot c)$ , a maximization of  $E$  also minimizes idle times. The corresponding SALB problem was hence labeled SALBP-E. Finally, the problem of finding a feasible balance for a given number of stations and a given cycle time is known as SALBP-F.

As already mentioned above, all four types of SALB are based on a set of limiting assumptions (c.f. Baybars, 1986; Scholl, 1999, ch. 2.2; Scholl and Becker, 2006):

- (S-1) Mass-production of one homogeneous product.
- (S-2) All tasks are processed in a predetermined mode (no processing alternatives exist).
- (S-3) Paced line with a fixed common cycle time according to a desired output quantity.
- (S-4) The line is considered to be serial with no feeder lines or parallel elements.
- (S-5) The processing sequence of tasks is subject to precedence restrictions.
- (S-6) Deterministic (and integral) task times  $t_j$ .
- (S-7) No assignment restrictions of tasks besides precedence constraints.
- (S-8) A task cannot be split among two or more stations.
- (S-9) All stations are equally equipped with respect to machines and workers.

Nearly all of these assumptions have been relaxed or somehow modified by various model extensions considered in the literature. This poses the question which, if any, of the assumptions of SALB is also constitutive for ALB in general.

With regard to real-world line balancing, one of the strongest simplifications of SALB is certainly caused by assumptions (S-2) and (S-9). In real-life, the same task might be performed in different modes and/or with different resources. More powerful machinery might for instance be purchased at higher cost, so that a time-cost trade-off influences the balancing decision. Under the limiting assumptions that processing modes are fixed for any task and each station is equally equipped an explicit consideration of *resources* (machine, men, material and/or tools) is not necessary. As a consequence the decision problem of SALB comes down to assigning tasks to stations. As soon as assumptions (S-2) and (S-9) have to be given up, line balancing needs also to decide on an assignment of required resources to stations along with the respective tasks. At the same time capacity-oriented objectives of SALB become insufficient as resource costs need to be considered.

In order to account for the large variety of ALB problems discussed in the literature, some SALB assumptions need to be modified whereas others are to be given up completely. The following set of assumptions (A-1) to (A-5) is considered to be constitutive for ALB in gen-

eral (SALB and other problem versions as well are contained as special cases with additional assumptions):

- (S-1) (A-1) The products to be manufactured (one or more) are known with certainty.
- (S-2) (A-2) A set of processing alternatives with all relevant attributes is given for each task.
- (S-3) (A-3) The line is to be configured such that target production quantities are satisfied for a certain planning horizon. This might be realized by setting the (average) cycle time(s) and, thus, production rate(s) or by seeking to produce as much as possible if maximum sales are not a limiting factor.
- (S-4) (A-4) The line flow is unidirectional.
- (S-5) = (A-5) The processing sequence of tasks is subject to precedence restrictions.
- (S-6) – (S-9) are given up.

Finally, this leads to the following definition of ALB. All optimization models which aim at supporting that part of assembly line configuration which deals with grouping tasks and their required resources to stations and are in line with the presented five assumptions (A-1) – (A-5) are considered to be assembly line balancing problems. All of those approaches are hence the subject of this work and will be included in the presented classification. An in-depth discussion of those approaches is provided in the following chapter.

### 3. Additional Characteristics of ALB

Due to very different conditions in industrial manufacturing, assembly line production systems and corresponding ALB problems show a great diversity. In the following, we characterize the relevant properties of assembly lines which have to be considered when balancing those lines. For other descriptions of assembly systems and different balancing problems we refer to, e.g., Buxey et al. (1973), Baybars (1986), Shtub and Dar-El (1989), Ghosh and Gagnon (1989), Erel and Sarin (1998), Scholl (1999, ch. 1), Rekiek et al. (2002b) as well as the most recent survey of Becker and Scholl (2006).

**Number and variety of products:** If only one product or several products with (almost) identical production processes, e.g. production of compact discs (Lebefromm, 1999, p. 3) or drinking cans (Grabau und Maurer, 1998), are assembled, the production system can be treated as a *single-model line*. In modern production systems, however, several products or different models of the same base product often share the same assembly line. In general, two different forms of organization are distinguished (c.f. Wild, 1972, p. 46; Buxey et al., 1973):

- A *mixed-model line* produces the units of different models in an arbitrarily intermixed sequence (cf. Scholl, 1999, p. 7). The sequence is important with respect to the efficiency of a line, because the task times may differ considerably between models. Therefore, the mixed-model ALB problem is connected to a *sequencing problem* which has to find a sequence of models to be produced such that inefficiencies like utility work, line stoppage,

and off-line repair are minimized (cf. Yano and Bolat, 1989; Sumichrast and Russel, 1990; Sumichrast et al., 1992; Bard et al., 1992; Tsai, 1995; Merengo et al., 1999). However, the balancing and the sequencing problem usually cannot be solved simultaneously, because the sequence depends on the short-term model-mix which is typically not known at the time when the line has to be balanced. Instead, the balancing problem is often based on an average model-mix. In order to anticipate the later sequencing problem adequately, a *horizontal balancing* objective is usually utilized which attempts to equalize the work content of stations over all models (cf. Scholl, 1999, ch. 3.3 and 3.4; Boysen, 2005, ch. B.2).

- A *multi-model line* produces a sequence of batches (each containing units of only one model or a group of similar models) with intermediate setup operations. Therefore, the ALB problem is not only connected to a (*batch*) *sequencing* but also to a *lot sizing problem* (cf., e.g., Burns and Daganzo, 1987, Dobson and Yano, 1994). However, both additional problems are typically not part of the long/medium-term configuration decisions.

**Line Control:** Assembly systems can be distinguished with regard to the control of job movements between stations. The exact type of line control has far reaching consequences for the structure of the balancing decision:

- Those assembly systems where a fixed time value restricts the work content of stations are referred to as *paced lines*. In the standard case, the same cycle time applies to all stations, so that they can all start their operations at the same time and workpieces are transferred at the same rate. These assembly lines thus have a fixed production rate, which is equal to the reciprocal of the cycle time. Under certain conditions, it might be required that cycle times are just kept on average (e.g. mixed-model lines, parallel line elements) or with a certain given probability (e.g. stochastic task times). Even locally diverging cycle times, which only apply to a group of stations, have been addressed, e.g. to meet different production targets in multi-model-production (Pinnoi and Wilhelm, 1997a; Gökçen and Erel 1998; Erel and Gökçen, 1999) or to enable the installation of test stations which need to examine reworked workpieces repeatedly (Lapierre and Ruiz, 2004).
- In *unpaced lines*, workpieces do not need to wait until a predetermined time span is elapsed, but are rather transferred when the required operations are finished. This type of line control is often implemented if stochastic variations influence processing times. Depending on the connection of the movements, two cases have to be distinguished (Buzacott and Shanthikumar, 1993):

In the *synchronous* case, workpieces are moved as soon as all stations have finished their operations. Stations which finish early must thus wait until the station with the highest work content has completed its operations (Lau and Shtub, 1987; Urban and Chiang, 2006b; Karabati and Sayin, 2003).

Under *asynchronous* movement, a station passes on its workpiece as soon as it has finished all operations as long as the successor is not *blocked* by another workpiece. It can hence continue to process the next workpiece, unless the predecessor was not able to deliver his workpiece in time (*starving*). In order to reduce waiting times due to blocking or

starving stations, typically buffer storages are installed along the line. Thus, the ALB is accompanied by the additional decision problem of *positioning and dimensioning buffers* (e.g., Buzacott, 1968, Suhail, 1983, Baker et al., 1990, Hillier and So, 1991, Hillier et al., 1993, Malakooti, 1994, Powell, 1994, Dolgui et al., 2002).

**Variability of task times:** In reality, task times are basically never deterministic (Tempelmeier, 2003).

- The assumption of *deterministic task times* is, however, justified whenever their expected variance is sufficiently small, as in case of simple manual tasks or highly reliable automated stations (cf. Johnson, 1983).
- In contrast to that, *stochastic task times* have to be considered if human work rate, skill and motivation and/or the failure sensitivity of complex processes are considerable (Buzacott, 1990, Robinson et al., 1990, Hillier and So, 1991, 1993, Pike and Martin, 1994).
- *Dynamic task times* which vary over time are, e.g., due to learning effects or successive improvements of the production process (e.g. Boucher, 1987, Chakravarty, 1988).

Task times are usually determined under some given standard situation. In case of deviations from these conditions, *task time increments* are possible (e.g. Buxey, 1974; Wilhelm, 1999; Bautista and Pereira, 2002; Scholl et al., 2006):

- Some stations might have non-standard conditions which lead to *station-dependent time increments* (e.g., additional transportation times of workpieces into a station (Bard, 1989), walking distances to change sides in a U-line (Sparling, 1998; Sparling and Miltenburg, 1998) or workers' return to the beginning of a station at the end of the cycle).
- Additionally, *sequence-dependent task time increments* may occur whenever the succession of operations influences their processing times. These might be caused by setup operations like tool swaps (Wilhelm, 1999) and repositioning of workpieces (Arcus, 1966; Bautista and Pereira, 2002). Furthermore, certain tasks might also transform the workpiece in such a way, that the execution of successive tasks is impeded. In automobile production such a relationship exists between installing the seat and the safety belt (Scholl et al., 2006).

**Line Layout:** Traditionally, an assembly line is organized as a *serial line*, where single stations are arranged along a (straight) conveyor belt. The actual line layout is, however, not necessarily determined prior to the balancing decision as long. The real-world arrangement of the conveyor belt does usually not affect the assignment decision and can thus be ignored. However, there are some extensions:

- In a *U-shaped assembly line* the stations are arranged along a rather narrow "U", where both legs are closely together. Stations in between those legs may work at two segments of the line facing each other simultaneously (crossover stations). This means, that workpieces can revisit the same station at a later stage in the production process without changing the

flow direction of the line. This can result in a better balance of station loads due to the larger number of task-station combinations (cf. Miltenburg and Wijngaard, 1994; Monden, 1998; Scholl and Klein, 1999). Instead of a single U, the line can also be organized as a sequence of several U-shaped line segments, called n-U line (cf. Miltenburg, 1998; Sparling, 1998).

- Other special line layouts might comprise *feeder lines*, which provide a main line with subassemblies (Hautsch et al., 1972; Lapierre and Ruiz, 2004). This can increase the complexity of the balancing problem considerably, when those feeder lines need to be balanced along with the main line.

**Parallelization of assembly work:** Assembly line production makes intensive use of increasing labour efficiency by partitioning the total work among different productive units. However, sometimes it is favourable to reduce the degree of division of labour by introducing some type of parallelization:

- Installing complete *parallel lines* each designed for one product or family of related products often allows better balances and increased productivity than mixing different products on the same line. The ALB is hence accompanied by additional decision problems concerning the number of lines to be installed and assigning products and work forces to lines (cf. Lehman, 1969; Geoffrion and Graves, 1976; Globerson and Tamir, 1980; Ahmadi et al., 1992, Balakrishnan and Vanderbeck, 1999).
- Even with a single line some advantages of parallelization can be utilized by installing *parallel stations*, i.e., the workpieces are distributed among  $p > 1$  stations which perform the same work content alternately thereby restricted by a (local) cycle time of  $p \cdot c$  time units (cf. Freeman and Jucker, 1967; Buxey, 1974; Pinto et al., 1981; Sarker and Shantikumar, 1983; Bard, 1989; Daganzo and Blumenfeld, 1994). As is the case with parallel lines, the equipment has to be installed for  $p$  times. Either physically separated units are constructed, which are served by a switching device, or parallel teams work on the same line section shifted by the cycle time (cf. Becker and Scholl, 2006). Although, there might be differences regarding space requirements, cost of capital and/or task time (Inman and Leon, 1994) no further differentiation between these two kinds shall be made, as they both only require to ensure that the average work content per cycle does not exceed the cycle time.
- Furthermore, individual tasks can be parallelized by assigning them to different stations of a serial line which cyclically perform them completely on alternating workpieces (*parallel tasks*, see Arcus, 1966, Pinto et al., 1975, Bukchin et al., 2002, Bukchin and Rabinowitch, 2005). The respective station times can then vary from cycle to cycle as long as they do not exceed the cycle time on average.
- If a workpiece is large enough to allow more than one operator or machine to work simultaneously as is, for instance, the case with automobiles, *parallel workplaces* can be installed in each station (Falkenauer, 2005). In this case, the balancing problem is connected to a detailed scheduling problem which has to avoid that different operators interfere with

each other. Usually, this is done by defining mounting places each of which can be occupied by a single workplace at a time only. A special case of a line with parallel workplaces is the *two-sided line* where each station consists of a left-hand-side and a right-hand-side workplace (Bartholdi, 1993; Kim et al., 2000a; Lee et al., 2001).

**Equipment and processing alternatives:** In order to perform a task assigned, the station must be equipped with productive *resources* like operators, machines and tools which provide the skills and/or technological capabilities required. Furthermore, the necessary material must be made available. We call the combination of these productive resources the *equipment* of a station.

- While in SALB it is assumed that all stations have multi-purpose equipments which are capable of carrying out any task, real-world assembly lines usually consist of differently equipped stations. So, the balancing problem is connected to an *equipment selection problem* which simultaneously has to decide on the equipment to be installed in each station. On the one hand, the equipment determines the efficiency of the balance obtainable and the resulting installation and operating costs. On the other hand, the combination of tasks within the same station load determines whether or not an equipment can be assigned to the respective station. This close interrelationship requires considering equipment selection and balancing simultaneously. In the literature this combined problem is referred to as the *assembly line design problem* (Baybars, 1986).
- Instead of explicitly considering the equipments required, different *processing alternatives* with differing task times, precedence relations or even alternative precedence subgraphs might be evaluated on a cost basis. The balancing problem is hence accompanied by a *process selection problem* (cf. Pinto et al., 1983). If, furthermore, the different processing alternatives also require different resources, then the equipment of a station is composed of those resources which are required to carry out the stations' work content. Instead of selecting a single equipment out of a set of predetermined equipment alternatives, here a stations' equipment is configured along with the task assignment. If several tasks require the same resources, *synergies* can be realized by combining these tasks into the same station loads, because the resources are needed only once and investment costs are at a minimum (cf. Boysen, 2005, pp. 105-108).
- With regard to *material supply*, the balancing decision may be closely related with several *logistic configuration problems*. For example, the extents and the volumes of material boxes including corresponding replenishment frequencies have to be determined (cf. Bukchin and Meller, 2005). Furthermore, these boxes have to be positioned along the line such that walking distances between workpieces and boxes are minimized.

**Assignment restrictions:** In ALB task assignments to stations are always restricted by precedence relationships. In model formulations, the corresponding precedence graph might either have a general structure or be restricted to some *special graph type*, e.g. linear (Kimms, 2000), diverging or converging graph. In any case, the precedence graph has to be (made) acyclic to find feasible task processing sequences (see Ahamdi and Wurgaft, 1994).

- In addition to precedence relations, special assignment restrictions might affect the grouping of certain tasks to stations. Often so called *zoning restrictions* are enforced, either to link a set of tasks which have to be assigned to the same station or to ensure that incompatible tasks are assigned to different stations (cf. Scholl, 1999, p. 12). Especially, if resources are not explicitly considered, zoning restrictions are a convenient way to make sure that tasks which require a very expensive resource share the same station (e.g. Dar-El and Rubinovitch, 1979). If a line is rebalanced and some machinery cannot be moved, a task can also be fixed to a certain station or might, on the contrary, be excluded from assignment, if the required resource is not present at a station (e.g. Kilbridge and Wester, 1961).
- Furthermore, the assignment of tasks to a station might be subject to constraints on the *cumulated* value of particular task attributes. For example, the space for placing machines or material boxes as well as for storing material or tools might be scarce such that station loads are only acceptable if the cumulated space requirement of assigned resources is not exceeding the available space (e.g., Kim and Park, 1995; Pastor et al., 2002; Sawik, 2002; Bautista and Pereira, 2006). Another constraint might concern the cumulated requirement on the operators' grip strengths which must not exceed a certain threshold from an ergonomic point of view (Carnahan et al., 2001). Besides these *upper bounds*, also *lower bounds* on cumulated values are possible, e.g., a minimum level of job variation (cf. Boyesen, 2005, p. 116).
- Especially, if workpieces undergo position changes while they are processed, task assignment might be restricted to stations of a certain *type*, where workpieces are in the required position (Johnson, 1983). In this case, the balancing decision can comprise the allocation of special machines, which lift or tilt the workpiece.
- The production process can also require the observance of *minimum* or *maximum distances* between tasks measured in time, space or sequence positions (see Pastor and Corominas, 2000; Rekiek and Delchambre, 2001). These distances can, e.g., allow an application to dry or to prevent melted metal from cooling down before a certain other task is carried out.

**Objectives:** Several of the extensions outlined above can only be considered in a meaningful way, if other objectives than the capacity-oriented ones introduced in section 2 are observed. Whenever alternative resources are available, resource costs will need to be regarded in the associated selection problem. Indeed, a wide variety of model extensions under cost or profit related objectives are proposed in the literature.

A *cost minimization* requires an assignment of cost types to elements of the assembly system, like tasks, stations, processing alternatives and/or resources. It is often difficult to determine the cost types unambiguously as the same cost values can be assigned to different cost units. Wage costs might, e.g., depend on the selected processing alternative, but can also be directly assigned to a task as long as no alternatives exist. If each station is equipped with a worker who can perform any task, wage costs might be directly assigned to stations. An in-depth

discussion of cost factors in ALB is provided by Steffen (1977) and Amen (1997, 2000a+b, 2001, 2006).

In order to *maximize profit*, which is defined as the difference of revenues and costs, not only the consumed resources (causing costs) have to be considered but also the output (generating revenues). The revenues depend on the sales prices, the production rate and, hence, the cycle time as well as the customer demands. Profit-oriented objectives are, e.g., considered by Zäpfel (1975), Rosenblatt and Carlson (1985), Martin (1994), Boysen and Fliedner (2006).

Instead or in addition to the objectives mentioned above, ALB can seek to achieve *smoothed station times*. In the literature, two different specifications are distinguished.

- In a mixed-model line it is desirable to smooth the variable work content of each station caused by the varying processing times of alternative models. This is also referred to as *horizontal balancing* (cf. Merengo et al., 1999).
- *Vertical balancing* aims at equating station times over all stations of the line for each model separately (Rachamadugu and Talbot, 1991) to avoid, among others, quality defects caused by stations with disproportionately large station times.

Furthermore, several authors propose to incorporate ergonomic aspects into the objectives, like grip strengths (Carnahan et al., 2001) or number of workpiece position changes (Bautista and Pereira, 2002) in order to keep physical stress for operators as low as possible or to optimize other measures of efficiency like product quality. Usually, those aspects are expressed by some sort of *composite score value*, e.g., in form of penalty costs.

Moreover, a combination of the presented objectives can be considered in some form of *multi-objective optimization* (e.g., Malakooti, 1991, 1994; Malakooti and Kumar, 1996; Gökcen and Agpak, 2006).

## 4. The Classification Scheme for ALB

In the preceding sections, we established our understanding of ALB in general and identified the main characteristics of problem extensions presented in the literature. Our approach to developing a classification scheme for ALB can hence be summarized as follows:

- The assumptions of SALB as outlined in section 2 are chosen as the common reference for classifying problem characteristics. That means, if not further stated it is supposed that the SALB assumptions apply, so that only deviations from SALB are explicitly provided.
- The basic classification scheme has been adopted from the widely accepted and successful classification scheme for machine scheduling of Graham et al. (1979). Another scheme which successfully helped structuring a complex research field is the one provided by Brucker et al. (1999) for project scheduling, which also employs the tuple-notation.
- As has been established in section 2, any ALB problem will at least consist of three basic elements: A *precedence graph* which comprises all tasks and resources to be assigned, the

*stations* which make up the line and to which those tasks are assigned and some kind of *objective* to be optimized. Accordingly, the presented classification will be based on those three elements which are noted as tuple  $[\alpha | \beta | \gamma]$ , where :

Precedence graph characteristics

Station and line characteristics

Objectives

One major advantage of the tuple-notation is that any default value, represented by the symbol  $\mathbf{O}$ , can be skipped when a tuple is actually specified. As explained above, the default values of all attributes constitute the SALB problem. In the following notation, the symbol \* always indicates that for the respective attribute the alternative values (except for  $\circ$ ) do not exclude each other and can be combined arbitrarily. As all attribute values are chosen such that they are unique, it is not necessary to specify the attribute designators within the tuples.

#### 4.1. Precedence Graph Characteristics

A precedence graph consists of nodes which represent the tasks of the production process and a set of arcs which represent the precedence relations between the tasks. Moreover, node and arc weights can be considered which reflect important attributes, like task times and processing alternatives, resource requirements and zoning restrictions. These precedence graph characteristics are represented by the six attributes  $\alpha_1$  to  $\alpha_6$ :

**Product specific precedence graphs:**  $\alpha_1 \in \{\text{mix, mult, } \mathbf{O}\}$

This attribute determines whether only a single product and, thus, a single precedence graph is considered or several ones have to be taken into account simultaneously. However, for the line balancing problem not the actual number of different products is decisive, but the degree of homogeneity of the precedence graphs.

$\alpha_1 = \text{mix}$  Varying models are manufactured on the same production system, the production processes of which are similar enough so that setup times are not present or negligible.

$\alpha_1 = \text{mult}$  Different products are manufactured in batches. Whenever another batch is to be processed, a setup occurs which requires time and resources.

$\alpha_1 = \mathbf{O}$  A single product is manufactured or the production processes of multiple products are (almost) identical so that they need not be distinguished.

**Structure of the precedence graph:**  $\alpha_2 \in \{\text{spec, } \mathbf{O}\}$

Some research papers restrict themselves to precedence graphs with special structure, mainly for developing more efficient specialized algorithms.

$\alpha_2 = \text{spec}$  The research is restricted to precedence graphs with some kind of special structure, e.g. linear, diverging or converging graphs.

$\alpha_2 = \mathbf{0}$  The precedence graph can have any acyclic structure.

**Processing times:**  $\alpha_3 \in \{t^{\text{sto}}, t^{\text{dy}}, \mathbf{0}\}^*$

In reality, processing times of tasks can vary in time, for instance due to complicated manual operations or defaults of machinery. ALB models can consider this phenomenon in different ways.

$\alpha_3 = t^{\text{sto}}$  Stochastic processing times: It is assumed that variations of processing times follow a known (or even unknown/partially known) distribution function.

$\alpha_3 = t^{\text{dy}}$  Dynamic variations of processing times: These variations are, e.g. due to learning effects of operators.

$\alpha_3 = \mathbf{0}$  Processing times are considered to be static and deterministic.

**Sequence-dependent task time increments:**  $\alpha_4 \in \{t_{\text{dir}}, t_{\text{ind}}, \mathbf{0}\}^*$

Such task time increments occur whenever the sequence of operations influences their processing times.

$\alpha_4 = \Delta t_{\text{dir}}$  If two tasks are done (in the same station) one directly after the other, additional time might be required for setup operations or tool changes.

$\alpha_4 = \Delta t_{\text{ind}}$  Indirect sequence-dependent time increments occur if the status achieved by completing particular tasks has an effect on the processing time of other tasks which are done later (in the same or another station).

$\alpha_4 = \mathbf{0}$  Sequence-dependent time increments are not considered.

**Assignment restrictions:**  $\alpha_5 \in \{\text{link, inc, cum, fix, excl, type, min, max}, \mathbf{0}\}^*$

Besides precedence relations, assignment restrictions might affect the grouping of tasks to station loads by forcing or forbidding certain combinations.

$\alpha_5 = \text{link}$  Subsets of tasks are linked such that these tasks must be assigned to the same station, e.g., because they employ the same resource which cannot be doubled.

$\alpha_5 = \text{inc}$  Subsets of tasks are incompatible and must not be assigned to the same station, e.g., because the tasks disturb each other (drilling and measuring) or require the workpiece in incompatible positions.

$\alpha_5 = \text{cum}$  The assignment of tasks to a station is subject to constraints on the cumulated value of particular task attributes (e.g., restricted space for storing material).

$\alpha_5 = \text{fix}$  Some tasks can only be assigned to particular stations, e.g., a required resource cannot be moved when reconfiguring a line.

$\alpha_5 = \text{excl}$  Some tasks must not be assigned to particular stations, e.g., because needed resources cannot be installed there.

$\alpha_5 = \text{type}$  Some tasks have to be assigned to a station from a certain type set, e.g., for working underneath a workpiece.

- $\alpha_5 = \min$  When assigning a task, minimum distances to other tasks have to be observed, e.g., for drying the paint.
- $\alpha_5 = \max$  Maximum distances between tasks have to be observed, e.g., because a required physical condition can only be maintained for a short time.
- $\alpha_5 = \mathbf{o}$  No assignment restrictions are considered.

**Processing alternatives**  $\alpha_6 \in \{\text{pa}^\lambda, \mathbf{Q}\}$ :

If processing alternatives exist, the production process and, hence the precedence graph is subject to change, so that an additional decision problem arises which is to select processing alternatives out of the set available.

- $\alpha_6 = \text{pa}^\lambda$  Processing alternatives can be distinguished according to the effects they have on the precedence graph by defining  $\lambda \in \{\mathbf{Q} \text{ prec, subgraph}\}$ :

$\lambda = \mathbf{Q}$ : Processing alternatives only deviate in processing times and costs (time-cost trade-off).

$\lambda = \text{prec}$ : The chosen alternative not only affects times and costs, but also precedence relationships between tasks. For example, an alternative placement of one component at the workpiece might change the precedence relations for installing another.

$\lambda = \text{subgraph}$ : Processing alternatives alter complete parts of the production process, so that whole subgraphs are substitutable. This might, e.g., occur whenever a set of options can either be installed separately or completely be replaced by a purchased module.

- $\alpha_6 = \mathbf{o}$  Processing alternatives are not considered.

**Remark:** Whenever processing alternatives are regarded, the aforementioned precedence graph characteristics (except for  $\alpha_1$  and  $\alpha_6$ ) might be related to alternatives and tasks rather than tasks only. This can be visualized by attaching <sup>pa</sup> to the respective attribute.

## 4.2. Station and line characteristics

Stations and their arrangement in the assembly line can be classified using the six attributes  $\beta_1$  to  $\beta_6$  of the second tuple entry  $\beta$ :

**Movement of workpieces:**  $\beta_1 \in \{\mathbf{Q}\vartheta, \text{unpac}^\lambda\}$

- $\beta_1 = \mathbf{Q}\vartheta$  In paced lines, a cycle time restricts the station time at each workstation with  $\lambda \in \{\mathbf{Q} \text{ each, prob}\}$  and  $\vartheta \in \{\mathbf{Q} \text{ div}\}$ :

$\lambda = \mathbf{Q}$ : The (average) work content per cycle of each station over all workpieces is restricted by the cycle time. While this restriction is strict in single-model-production, it has just to be fulfilled on average in the case of mixed-model production, some type of parallelization and/or stochastic task times.

$\lambda = \text{each}$  : Each single model strictly must fulfill the cycle time restriction in mixed- or multi-model production.

$\lambda = \text{prob}$  : The cycle time restriction is obeyed with a given probability (i.e. stochastic task times) or proportion (i.e. uncertain model-mix).

$\vartheta = \mathbf{O}$  : All stations and models have to regard the same global cycle time.

$\vartheta = \text{div}$  : (Local) cycle times diverge between stations or models.

$\beta_1 = \text{unpac}^\lambda$  An unpaced line is not strictly restricted by a cycle time. Instead, it advances when stations have completed their tasks; with  $\lambda \in \{\mathbf{O}, \text{sync}\}$ :

$\lambda = \mathbf{O}$  : Asynchronous: As soon as a station completes its work, the workpiece is moved to the next station or a buffer in front of this station unless blocking occurs.

$\lambda = \text{sync}$  : In synchronous lines, the movement of workpieces is coordinated between stations. The workpieces are transferred to the respective next station when all stations have completed their current workpiece.

**Line layout:**  $\beta_2 \in \{\mathbf{O}, \text{u}^\lambda\}$

$\beta_2 = \mathbf{O}$  The stations are arranged in a serial manner along the flow of the line.

$\beta_2 = \text{u}^\lambda$  The U-shaped line layout with crossover stations is used; with  $\lambda \in \{\mathbf{O}, \text{n}\}$ :

$\lambda = \mathbf{O}$  : The line forms a single U.

$\lambda = \text{n}$  : An n-U line composed of multiple U-shaped segments is considered.

**Remark:** If other special layouts will be introduced, the classification can easily be extended.

**Parallelization:**  $\beta_3 \in \{\text{pline}^\lambda, \text{pstat}^\lambda, \text{ptask}^\lambda, \text{pwork}^\lambda, \mathbf{O}\}^*$

$\beta_3 = \text{pline}^\lambda$  More than one parallel line is to be balanced or the number of lines installed is part of the decision problem.

$\beta_3 = \text{pstat}^\lambda$  When stations are parallelized, their resources and work contents are duplicated so that they process all assigned tasks alternately.

$\beta_3 = \text{ptask}^\lambda$  A parallelized task is assigned to more than one station. In addition to their regular work content, stations process the parallelized task interchangeably.

$\beta_3 = \text{pwork}^\lambda$  Several working places simultaneously work on the same workpiece at different mounting places such that they do not obstruct each other.

$\beta_3 = \mathbf{O}$  Neither type of parallelization is considered.

**Remark:** If a certain fixed or maximum level of parallelization is constitutive for the problem, this is visualized by the superscript  $\lambda \in \{2, 3, \dots\}$ . Such a constraint might be due to technological or space restrictions. For example, the stations of a two-sided line consist of two parallel working places, i.e.,  $\beta_3 = \text{pwork}^2$ . If the level of parallelization is unconstrained or can be set to an arbitrary value by the planner,  $\lambda = \mathbf{O}$  is used.

**Resource assignment:**  $\beta_4 \in \{\text{equip}, \text{res}^\lambda, \mathbf{0}\}^*$

Typically, multiple resources, forming the station equipment, are necessary to perform tasks at stations.

$\beta_4 = \text{equip}$  For each station exactly one equipment is to be chosen out of a set of prespecified equipment alternatives.

$\beta_4 = \text{res}^\lambda$  The tasks assigned to a station determine the set of resources which need to be allocated there. If multi-usage of resources is possible, different types of synergies can be distinguished by  $\lambda \in \{\mathbf{0}, 01, \text{max}\}^*$ :

$\lambda = 01$ : If more than one task can be performed on the same resource (a tool or machine), investment cost is reduced if those tasks are assigned to the same station, because the resource needs to be installed only once. That is, a 0-1 investment decision has to be made for each station-resource combination (install if any task requires the resource, do not install else).

$\lambda = \text{max}$ : If tasks differ with respect to the resource quality (concerning, e.g., speed, capabilities, qualification) they require, the resources to be selected such that they fulfill the maximum demand level. For example, if operators require a specific qualification to perform (difficult) tasks, the most challenging task assigned to a station defines the needed qualification level of the operator and, hence, the wage costs to be paid at that station.

$\lambda = \mathbf{0}$ : Resources are modeled explicitly to account for another type of synergy and/or dependency.

$\beta_4 = \mathbf{0}$  Resources are not considered explicitly but might influence the balancing decisions via assignment restrictions, processing alternatives etc. implicitly.

**Remark:** If resources are modeled explicitly, a part of the task-station assignment restrictions ( $\alpha_5$ ) might be replaced by *task-resource* and *resource-station* assignment restrictions.

**Station-dependent time increments:**  $\beta_5 \in \{\Delta t_{\text{unp}}, \mathbf{0}\}$

$\beta_5 = \Delta t_{\text{unp}}$  Some part of the station time is consumed by unproductive activities, e.g. workpiece transportation.

$\beta_5 = \mathbf{0}$  Station-dependent time increments are not regarded.

**Additional aspects of line configuration:**  $\beta_6 \in \{\text{buffer}, \text{feeder}, \text{mat}, \text{change}, \mathbf{0}\}^*$

Depending on the production system, additional technical requirements might need to be considered for balancing the line:

$\beta_6 = \text{buffer}$  Buffer storages are required and have to be allocated and dimensioned.

$\beta_6 = \text{feeder}$  One or more feeder lines which flow into a main line necessitate a simultaneous coordination of task assignments and cycle time.

- $\beta_6 = \text{mat}$  Boxes which contain the required material need to be positioned and dimensioned.
- $\beta_6 = \text{change}$  If certain tasks require the workpiece to be in a particular position (lifted, tilted, etc.), a decision has to be made, whether positions are fixed within stations or special machines are assigned to stations which allow for a change of positions.
- $\beta_6 = \mathbf{o}$  No additional aspects of line configuration are regarded.

### 4.3. Objectives

Finally, the optimization of ALB will be guided by some objective which evaluates solutions. In the case of multi-objective optimization more than a single objective can be selected out of the set  $\gamma \in \{m, c, E, \text{Co}, \text{Pr}, \text{SSL}^\lambda, \text{score}, \mathbf{o}\}^*$ .

- $\gamma = m$  Minimize the number of stations  $m$  subject to a given output target for a certain planning horizon (specified by the cycle time  $c$  or the production rate). In case that parallel stations or workplaces are included in  $m$ , the number of sequential stages might be restricted (maximal line length).
- $\gamma = c$  Minimize the cycle time  $c$  or, equivalently, maximize the production rate subject to a given number of stations  $m$ .
- $\gamma = E$  Maximize the line efficiency  $E$  (productive part of the line capacity) such that restrictions on the production rate and/or the number of stations are fulfilled.
- $\gamma = \text{Co}$  Cost minimization for a given output target. A classification of cost factors shall not be provided as cost types can be assigned to various elements of the assembly system depending on the considered model formulation.
- $\gamma = \text{Pr}$  The profit, which is defined as the difference between the revenue (which depends on the production rate and, hence, the cycle time) and the costs, is maximized. A further distinction shall not be provided because of the aforementioned issue of cost type assignment.
- $\gamma = \text{SSL}^\lambda$  Station times are to be smoothed with  $\lambda \in \{\text{stat}, \text{line}\}$ :
- $\lambda = \text{stat}$  : In mixed-model production the varying station times caused by the different models are to be smoothed (horizontal balancing).
- $\lambda = \text{line}$  : Station times are balanced over all stations of the line (vertical balancing).
- $\gamma = \text{score}$  The objective is to minimize or maximize some composite score which is related to one or more attributes describing bottleneck aspects or further measures of efficiency, e.g. required grip strengths, quality or number of workpiece position changes.
- $\gamma = \mathbf{o}$  No objective function is required, only feasible solutions are searched for.

## 5. Classifying the Literature

With this scheme on hand, we classify that part of the ALB literature which deals with GALB. For the versions of SALB, we only specify the tuple-notation, because there is no need to structure this field of research (instead, we refer to Scholl and Becker 2006):

SALBP-F: [ | | ]                      SALBP-1: [ | | m ]  
 SALBP-2: [ | | c ]                      SALBP-E: [ | | E ]

The following table assigns the tuple-notation to each relevant publication and its contribution to the research field of GALB. If more than one problem version is treated in a paper the most complex tuple-notation is reported. Further we distinguish between the following contributions made by the respective publication:

M	mathematical model	E	exact solution procedure
B	bound computation	S	simulation approach
HS	heuristic start procedure for initial solution, mostly priority rule based	HI	heuristic improvement procedure
SA	sensitivity / stability analysis	HM	meta-heuristic

**Remark:** The following list does not include the numerous papers which treat related decision problems of configuration planning which assume a *given* line balance. For example, a massive body of literature covers the buffer allocation problem in unpaced asynchronous lines (see the review papers of Dallery and Gershwin, 1992; Papadopoulos and Heavey, 1996; Gershwin, 2000). Since a change in station loads typically affects optimal buffer allocation considerably, a simultaneous optimization would be desirable to identify the systems' global optimum. However, such simultaneous approaches are currently not on-hand.

Publication	Notation	Contribution
Aase et al. (2003, 2004)	[   u   m ]	M, B, E, S
Agnetis et al. (1995)	[spec, inc, fix     SSL <sup>line</sup> ]	E
Agrawal (1985)	[     score]	HS
Ajenblit and Wainwright (1998)	[   u   m, SSL <sup>line</sup> ]	HM
Akagi et al. (1983)	[   pwork   ]	HS
Amen (1997, 2000a+b, 2001, 2006)	[   res <sup>max</sup>   Co]	M, B, E, HS
Arcus (1966)	[mix, t <sub>dir</sub> , cum, fix   res <sup>max</sup> , t <sub>unp</sub> , pwork   E]	HS
Askin and Zhou (1997)	[mix   pstat   Co]	M, B, HS
Bard (1989)	[   pstat, ptask, t <sub>unp</sub>   Co]	E
Bartholdi (1993)	[fix, type   pwork <sup>2</sup>   m]	HS
Bautista and Pereira (2002)	[inc, t <sub>dir</sub>     m, score]	HM
Bautista and Pereira (2006)	[cum     m, c, score]	M, HI, HM
Bautista et al. (2000)	[inc     m, score]	HS, HM
Baykasoglu and Özbakir (2006)	[t <sup>sto</sup>   prob, u   m]	HM
Berger et al. (1992)	[spec   u   m]	B, E
Boysen and Fliedner (2006)	[t <sup>sto</sup> , link, inc, cum, pa   u, pstat, ptask, res <sup>01</sup> , res <sup>max</sup>   Pr]	HS, HM, E
Bukchin and Rabinowitch (2005)	[mix   div, ptask, res <sup>01</sup>   Co]	M, B, E

Publication	Notation	Contribution
Bukchin and Rubinovitz (2003)	[pa pstat, equip Co]	M, B, E
Bukchin and Tzur (2000)	[pa equip Co]	M, B, E
Bukchin et al. (1997), Bukchin and Masin (2004)	[  pwork m, score]	B, E
Bukchin et al. (2002)	[mix ptask score]	M, HS, HI
Buxey (1974)	[ t <sub>dir</sub> , link, inc, max pstat score]	HS
Capacho and Pastor (2004)	[pa <sup>subgraph</sup>    m]	M
Carnahan et al. (2001)	[cum   c, score]	HS, HM
Carraway (1989)	[t <sup>sto</sup>  prob m]	E
Carter and Silverman (1984)	[t <sup>sto</sup>    Co]	HS
Chakravarty (1988)	[t <sup>dy</sup>    E]	HS, E
Chakravarty and Shtub (1985)	[mult div Co]	M, HS
Chakravarty and Shtub (1986)	[mult, t <sup>sto</sup>  div Co]	HS
Chiang and Urban (2002)	[t <sup>sto</sup>  prob m]	HS, HI
Dar-El and Rabinovitch (1988)	[mult, t <sup>dy</sup>    Co]	M
Deckro (1989)	[link, inc, max   m, c]	M
Dolgui and Ihnatsenka (2004), Dolgui et al. (1999, 2001a+b+c, 2003, 2006)	[link, inc pwork Co]	M, HS, B, E
Domschke et al. (1996)	[    E, SSL <sup>stat</sup> ]	E, S
Erel and Gökçen (1999), Gökçen and Erel (1998)	[mix div m]	M, E
Erel et al. (2001)	[  u m]	HM
Erel et al. (2005)	[t <sup>sto</sup>  u Co]	HS
Gamberini et al. (2004)	[t <sup>sto</sup>  u Co, score]	HS
Gökçen and Agpak (2006)	[  u m, c, score]	M
Gökçen and Erel (1997)	[mix, link, inc   score]	M
Gökçen et al. (2005)	[  u m]	M, E
Haq et al. (2005)	[mix   m]	HS, HM
Hautsch et al. (1972)	[link pwork <sup>2</sup> , res <sup>max</sup> , feeder E]	HS
Henig (1986)	[t <sup>sto</sup>  prob Co]	E
Johnson (1983)	[type div, ptask m]	B, E
Johnson (1991)	[type   m]	B, E
Kao (1976, 1979)	[t <sup>sto</sup>  prob m]	E
Karabati and Sayin (2003)	[mix unpac <sup>sync</sup>  score]	M, HS
Kim and Park (1995)	[cum equip m]	M, HS, E
Kim et al. (2000a)	[fix, type pwork <sup>2</sup>  m]	HM
Kim et al. (2000b+c, 2006)	[mix u SSL <sup>line</sup> ]	HM
Kimms (2000)	[mult, spec unpac, equip Co]	M, B, E
Klenke (1977)	[t <sup>sto</sup> , link, inc prob Pr]	M, E
Kottas and Lau (1973, 1976, 1981)	[t <sup>sto</sup>    Co]	HS
Lapierre and Ruiz (2004)	[link, inc, type div, pwork <sup>2</sup> , feeder m]	HS
Lapierre et al. (2006)	[type pwork <sup>2</sup>  m]	HM
Lee et al. (2001)	[type pwork <sup>2</sup>  score]	HS
Leu et al. (1994)	[fix   score]	HS, HM
Levitin et al. (2006)	[pa equip c]	HM
Lyu (1997)	[t <sup>sto</sup>    Co]	HS
Macaskill (1972)	[mix   E]	HS

Publication	Notation	Contribution
Malakooti (1991)	[     m, c, Co]	M, E
Malakooti (1994)	[   unpac, buffer   m, c, Co, score]	HS
Malakooti and Kumar (1996)	[fix   unpac, buffer   m, c, Co, score]	HS
Matanachai and Yano (2001)	[mix     SSL <sup>line</sup> , SSL <sup>stat</sup> ]	HS, HI
McMullen and Frazier (1997, 1998), McMullen and Tarasewich (2003)	[mix, t <sup>sto</sup>   pstat   Co, SSL <sup>line</sup> , score]	HM
Merengo et al. (1999)	[mix   each   m, SSL <sup>line</sup> , SSL <sup>stat</sup> ]	HS, HI
Miltenburg (1998)	[fix   u <sup>n</sup> , t <sub>unp</sub>   m, score]	E
Miltenburg (2002)	[mix   u   SSL <sup>line</sup> , SSL <sup>stat</sup> ]	HM
Miltenburg and Wijngaard (1994)	[   u   m]	E, HS
Miralles (2005)	[pa, link, cum   equip   c]	M, B, E
Moodie and Young (1965)	[t <sup>sto</sup>     SSL <sup>line</sup> ]	HS
Nicosia et al. (2002)	[pa   equip   Co]	B, E
Nkasu and Leung (1995)	[t <sup>sto</sup>   prob   m, c, E]	S
Park et al. (1997)	[spec, inc, pa <sup>prec</sup>     c]	HS, HI
Pastor and Corominas (2000)	[link, inc, type, max     SSL <sup>line</sup> ]	M, HS, HM
Pastor et al. (2002)	[mult, cum, fix     c, SSL <sup>line</sup> , SSL <sup>stat</sup> ]	HS, HM
Pinnoi and Wilhelm (1997a)	[mult, link, inc, cum, type, pa   div, equip, pstat, pwork   Co]	M
Pinnoi and Wilhelm (1997b)	[     SSL <sup>line</sup> ]	M, B, E
Pinnoi and Wilhelm (1998)	[pa   equip   Co]	M, B, E, HS
Pinto et al. (1975)	[   ptask   Co]	M, B, E
Pinto et al. (1981)	[   pstat   Co]	M, B, E
Pinto et al. (1983)	[pa     Co]	M, B, E
Ponnambalam et al. (2000)	[     m, E, SSL <sup>line</sup> ]	HM
Rachamadugu and Talbot (1991)	[     SSL <sup>line</sup> ]	M, HI
Raouf and Tsui (1982)	[t <sup>sto</sup> , fix, excl   prob   m, SSL <sup>stat</sup> ]	HS
Reeve and Thomas (1973)	[t <sup>sto</sup>     SSL <sup>line</sup> ]	HS
Rekiek et al. (2001)	[ t <sub>dir</sub> , link, inc, fix, type, pa     Co, SSL <sup>line</sup> ]	HM
Rekiek et al. (2002a)	[link, fix, pa     Co, SSL <sup>line</sup> ]	HM
Roberts and Villa (1970)	[mix, link   each, ptask   m]	M, E
Rosenberg and Ziegler (1992)	[   res <sup>max</sup>   Co]	M, B, HS
Rosenblatt and Carlson (1985)	[     Pr]	M, HS, E
Rubinovitz and Bukchin (1993)	[pa   equip   m]	B, E
Sabuncuoglu et al. (2000)	[     m, SSL <sup>line</sup> ]	HM
Sarin and Erel (1990)	[t <sup>sto</sup>     Co]	E
Sarin et al. (1999)	[t <sup>sto</sup>     Co]	HS, HI
Sarker and Shantikumar (1983)	[   pstat   Co, SSL <sup>line</sup> ]	HS, HI
Sawik (2002)	[mix, cum   div, ptask   c]	M
Schofield (1979)	[mix, link, inc, fix, excl   t <sub>unp</sub>   c]	HS
Scholl and Becker (2005)	[   res <sup>max</sup>   Co]	M, B, E,
Scholl and Klein (1999)	[   u   E]	B, E
Scholl et al. (2006)	[   t <sub>ind</sub>   m]	M, E
Shin and Min (1991)	[t <sup>sto</sup>   prob   Co]	HS
Shtub (1984)	[t <sup>sto</sup> , pa     Co]	M, HS
Shtub and Dar-El (1990)	[     m, c, score]	M, HS
Silverman and Carter (1986)	[t <sup>sto</sup>     Co]	HS
Sniedovich (1981)	[t <sup>sto</sup>   prob   m]	E

Publication	Notation	Contribution
Sotskov et al. (2006)	$[t^{sto}   m]$	SA
Sparling (1998)	$[   u^n, pline, t_{unp}   m]$	M, HS
Sparling and Miltenburg (1998)	$[mix   u, t_{unp}   m, SSL^{stat}]$	M, HS
Sphicas and Silverman (1976)	$[t^{sto}   prob   m]$	M
Süer (1998)	$[   pline, pstat   m, c, score]$	HS
Suresh and Sahu (1994), Suresh et al. (1996)	$[t^{sto}     SSL^{line}, score]$	HM
Thomopoulos (1970)	$[mix     m, SSL^{stat}]$	HS
Tsujimura et al. (1995)	$[t^{sto}     score]$	M, HM
Ugurdag et al. (1997)	$[     c, SSL^{line}]$	M, HS, HI
Urban (1998)	$[   u   m]$	M
Urban and Chiang (2006a)	$[t^{sto}   prob, u, t_{unp}   m]$	M, B
Urban and Chiang (2006b)	$[t^{sto}   unpac^{async}   m]$	HS, HM
Van Hop (2006)	$[mix, t^{sto}   div   m]$	M, HS, HI
Vilarinho and Simaria (2002)	$[mix, link, inc   prob, pstat   m, SSL^{line}, SSL^{stat}]$	M, HM
Visich et al. (2002)	$[mix   u   SSL^{stat}]$	HM
Wilhelm (1999)	$[   t_{dir}, pa   equip   Co]$	M, B, E
Wilson (1986)	$[t^{sto}, pa     Co]$	M
Zäpfel (1975)	$[link, inc     Pr]$	M, E

## 6. Research Challenges and Conclusions

The classification of the literature reveals a number of important open fields of research which require an in-depth discussion to narrow the gap between research and practice. In the following, we discuss those which seem to be most important.

Within the lifetime of a modern production system, ALB problems do not only occur once prior to its construction, but rather continuously as *reconfiguration* or *rebalancing* (Schofield, 1979, Falkenauer, 2005). These might be necessary to react on shifts in the demand structure or whenever new production technologies become available. As a consequence, stations will often have a unique identity expressed by a position in the workshop, assigned resources or space restrictions ( $\alpha_5 = cum$ ), instead of being abstract entities. In the literature, reconfiguration is often considered by introducing assignment restrictions. If machinery is too heavy for being moved to another station, the assignment of tasks which require such a machine can be forced to its current station by an assignment restriction ( $\alpha_5 = fix$ ). Nevertheless, often the reallocation of machines is not technically impossible, but rather associated with moving costs (Gamberini et al., 2004). The same applies to operators at a station, who might need to be trained with regard to the new work content. Costs can be decreased when workers keep as much of their previous tasks as possible. Accordingly, assembly line balancing should regard these *moving costs*.

A similar phenomenon occurs when several tasks require the same resources. In this case, cost of capital can be decreased when tasks are assigned to the same station, which is typically enforced by zoning restrictions ( $\alpha_5 = link$ ). Whether the benefits associated with avoid-

ing an extra resource actually surmount the disadvantages of a more restricted balancing problem, can just be answered by explicitly modeling *cost synergies* ( $\beta_4 = \text{res}^{01}$ ).

In the literature, these cost aspects are often neglected and costs are directly assigned to tasks or stations instead. This, however, does often not reflect the real situation. In real life, costs only arise by the purchase or usage of a resource, which is positioned at a station. By exactly assigning costs to the resources instead of stations or tasks, a more realistic modeling of the decision problem could be attained.

*Sequence-dependent relations* between tasks also require further investigation. Occasionally, precedence-dependent time increments (Buxey, 1974; Wilhelm, 1999; Rekiek et al., 2001; Bautista and Pereira, 2002) are regarded, which result from a tool or position change between predecessors and their direct successors ( $\alpha_4 = \Delta t_{\text{dir}}$ ). However, task times can also be influenced by an indirect succession of tasks ( $\alpha_4 = \Delta t_{\text{ind}}$ ). These time increments occur if the resulting transformation of a completed task obstructs the implementation of another, e.g. seat and safety belt of a car. These dependencies can even cover more than two tasks. For example, the assembly of a car's hand-break might be obstructed only after both front seats have been installed. Otherwise, the access is free from at least one side. In an extreme case, the assembly of a hand break might even be impossible if both seats are installed. These phenomena cannot be included in traditional arcs of the precedence graph, since they depict dependencies exclusively between two tasks. By disregarding these dependencies assembly line balancing might result in infeasible line balances.

Little attention has been paid to the so called "additional aspects of line configuration". Real world assembly lines often have one or more *feeder lines* ( $\beta_5 = \text{feeder}$ ) which flow into a main line (Tempelmeier, 2003). A possible approach to configure such an assembly system is to balance the main line first and use the retrieved cycle time to balance each feeder line separately. Whether this decomposition results to a global optimum of the whole assembly system is questionable, especially if operators can perform tasks on both, the feeder and the main line, at their contact point (Lapierre and Ruiz, 2004).

Another important but almost completely ignored aspect is the *material supply* of an assembly line ( $\beta_5 = \text{mat}$ ). Material is usually provided in some kind of container along the line. A container could possibly be some traditional shelf or box, but also an automated guided vehicle (AGV) which delivers the material directly to the stations. In such a setting, new decision problems arise concerning the dimensions of these containers for different components (Bukchin and Meller, 2005) and their allocation on the line. If more than one station is served by the same material container or if distances vary depending on the actual position of the operator, station times are affected by variable walking distances for fetching the material. Due to these interdependencies, it might be necessary in practice to solve line balancing and material supply problems simultaneously.

Furthermore, little attention has been paid to *parallel working places* ( $\beta_3 = \text{pwork}$ ). Although, simultaneous work at a station requires a sufficiently large workpiece, there are important practical applications in automobile and related industries (Falkenauer, 2005). Work-

ers are able to perform operations at different mounting places at the same time if tasks are compatible and do not require the workpiece to be in an exclusive position. In case of parallel work, precedence constraints lead to additional idle times whenever one operator has to wait for another one finishing a preceding task. Accordingly, the station time can no longer be computed by simply summing up the assigned task times, but requires the solution of a detailed scheduling problem instead. To close the gap to practice in this field of research, optimization models have to be developed which go far beyond a simple assignment of tasks to working places.

Over the last few years, a lot of research has regarded *processing alternatives*. Most tasks can be performed in different ways: A faster machine and/or multiple manning decrease the processing time but commonly raise cost of capital (time-cost trade-off). Choosing processing alternatives and balancing the line simultaneously promises better configurations than deciding successively. A successive planning approach selects a processing alternative for each task first and then balances the line with regard to the chosen alternatives. Hints on the advantage's extent when planning simultaneously (maybe with less efficient algorithms) in comparison to planning successively would be of great value in this field of research.

Two different approaches have been proposed to incorporate processing alternatives into ALB. The first is known as the *equipment selection problem* [pa|equip|Co] (e.g. Pinnoi and Wilhelm, 1998; Bukchin and Tzur, 2000). It is based on the assumption that there is a fixed set of equipments (complete set of resources) exactly one of which has to be selected and assigned to a station. The alternative approach consists in *assigning processes to tasks*. In addition to line balancing, for each task exactly one processing alternative has to be chosen out of a set of possible ones. Because these processes require resources, the problem can be interpreted as an (implicit) *equipment design problem*, represented by [pa| |Co] in our classification scheme (e.g. Pinto et al., 1983). The chosen processes are usually not independent of each other, which has to be reflected by considering possible synergies arising from jointly using resources for several tasks at a station ( $\beta_4 = \text{res}^{01}$ ) or different types of assignment restrictions ( $\alpha_5 = \text{link}^{\text{pa}}$ ,  $\alpha_5 = \text{inc}^{\text{pa}}$ ,  $\alpha_5 = \text{cum}^{\text{pa}}$ ).

Both concepts can in principle be used interchangeably. However, the resulting mathematical models might differ considerably. While the equipment selection approach excludes incompatible processing alternatives prior to modeling at the cost of a potentially large number of equipment alternatives, the equipment design approach needs to exclude incompatible combinations within the model which can hence lead to more restrictions and a higher complexity. Due to limitations in data generation and model solving, both approaches have to be limited to a subset of possible alternatives in real-world settings. In order to find out which approach is better suited to different situations, comparative analyses should be performed.

In the literature, both approaches are so far restricted to covering different modes of accomplishing a task. In reality, processing alternatives might, however, also alter precedence constraints between tasks ( $\alpha_6 = \text{pa}^{\text{prec}}$ ). For example, the automated assembly of an option using a robot can be obstructed by an already installed part whereas a manual assembly is still pos-

sible. Even whole subgraphs of the precedence graph may be substituted by a processing alternative ( $\alpha_6 = \text{pa}^{\text{subgraph}}$ ). An example is the assembly of a procured module which unites a set of optional features instead of individually installing each option. Both approaches of processing alternatives need to be extended in this direction.

Our survey finally revealed that there is a dire need of systematic evaluations to identify those decision concepts – combining a (theoretical) problem type considered, the mathematical model derived and the (exact or heuristic) solution method applied – which are best suited to solving real-world assembly line balancing problems. In particular, this includes the systematic generation of realistic test beds and the development of solution algorithms flexible enough to jointly cover as many problem characteristics as possible. The structure provided in this paper might therefore also be employed to develop new instance generators and solution procedures which cover all or a certain systematic subset of important problem characteristics.

We would thus like to encourage all researchers to make use of the provided scheme (and to extend it if necessary), in order to establish a common reference for past as well as future publications concerning assembly line balancing. This should ease the coordination of remaining steps to close the stated gap between research and practice substantially.

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